SUPPLEMENTARY MATERIALS: A MATHEMATICAL FRAMEWORK FOR DEVELOPING FREEZING PROTOCOLS IN THE CRYOPRESERVATION OF CELLS

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Abstract. We provide quantitative verification of the second-order convergence of the numerical scheme outlined in Appendix B.

1. Quantitative verification.

For a quantity X, we define the relative error as

$$\max_{t>0}\left|\frac{X(t)-X_\infty(t)}{X(t)}\right|, \tag{1}$$

where X_∞ is our estimate of the 'exact solution'. We are interested in quantifying the relative error as the mesh size varies. We show the relative error in the concentration of CPA at the cell membrane, the concentration of CPA at the ice-water interface, the position of the cell membrane, and the position of the ice-water interface in Figure 1, for a varying mesh size. The convergence appears to be second-order, as predicted by our numerical scheme.

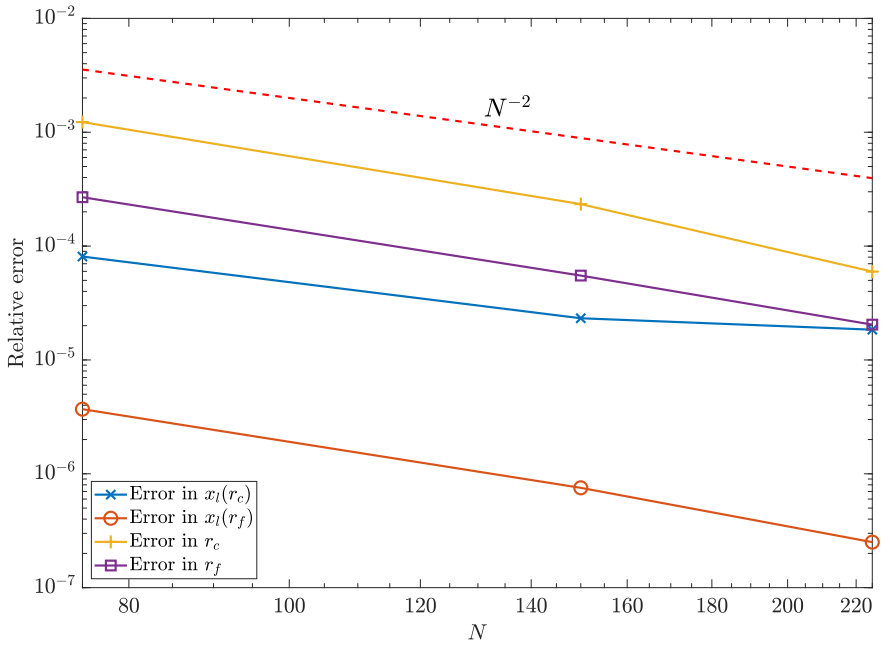

FIG. 1. *The relative error in the concentration of CPA at the cell membrane, the concentration of CPA at the ice-water interface, the position of the cell membrane, and the position of the ice-water interface, as the mesh size varies, with a cooling rate of* 0.1 $K\ s^{-1}$. *We use (20, 75), (40, 150), and (60, 225) grid points, where the first number in brackets corresponds to the number of grid points in the ice and cell phases, and the second corresponds to the number of grid points in the extracellular liquid medium region. The x-coordinate in the figure denotes the number of grid points in the liquid region,* i.e. *the second number in brackets above. The dashed red line shows the scaling* N^{-2}, *representing second-order convergence.*

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